Backstepping in infinite dimensional for the time fractional order partial differential equations

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Abstract
This paper focuses on the application of backstepping control scheme for the time fractional order partial differential equation (FPDE). The fractional derivative is presented by using Caputo fractional derivative. The design technique here can exhaust systems with an arbitrary finite number of open loop unstable eigenvalues and is not limited to a certain kind of boundary actuation. We show how the FPDE is converted into a Mittag-Leffler stability by designing invertible coordinate transformation. Numerical simulation is given to demonstrate the effectiveness of the proposed control scheme.

Keywords: Boundary control, backstepping, stabilisation, coordinate transformation, fractional order PDE

INTRODUCTION

We can define the boundary control as a distributed parameter control that has been widely studied and developed in the control theory. It is not very recent when the researchers started to investigate the general parabolic equations and the boundary feedback stabilisation of this kind of equations. Triggiani [1] and Lasiecka [2] used the semi group theory in order to evaluate a general form to obtain the eigenvalues for parabolic problems. Then, an auxiliary functional observers has been developed by Nambu [3], to stabilise diffusion equations by the use of boundary observation and feedback. Moreover Bensoussan et al. [4] discussed the stabilisation of the optimal control setting by the boundary control.

There is a huge amount of attention these days on the boundary control of K-S equation [5,6], Burgers equation [7], KdV-Burgers equation, C-H equation [8,9], heat equation [10-13]. The boundary control of the K-S equation is studied in [5] with an external excitation, specifically by using the semi group theory and Banach contraction fixed point to satisfy the existence and uniqueness of the solution. It has been presented in [7] the existence of an optimal controller and suitable index of performance $J(y; u)$ regarding to Galerkin method. While in [14], a new simple controller was proposed for Chen’s chaotic system.

The heat equation is a typical parabolic equation, which has rich physics background. Recently, many researchers have been focusing on the heat equation with backstepping control law [15-19], which is still a boundary control. Nevertheless, according to our knowledge, there are only few attempts on the method of boundary feedback stabilisation to deal with the unstable FPDE. The boundary stabilisation of fractional wave equation based on numerical solution technique has given a boundary control of a Caputo fractional wave equation through a fractional order boundary controller was studied in [20,21].

One important feature of the fractional order models over the integer order ones is that many real life applications can be described by utilising notation of fractional order [22,23].

Based on what was listed above, this paper focuses on the following FPDE

$$\begin{align*}
\frac{\partial}{\partial t}^q s(x,t) &= \frac{\partial}{\partial x}^q s(x,t) + \gamma(x)s(x,t) & \text{in } (0,1) \times (0,\infty), \\
\text{where the boundary conditions are} & \\
s(0,t) &= 0, \quad t > 0, \\
or \\
s_x(0,t) &= 0, \quad t > 0, \\
s(1,t) &= \int_0^1 k(\xi)s(x,\xi)d\xi, \\
\text{and} & \\
\text{where, } 0 < q \leq 1, \gamma > 0, \gamma(x) \in L_2(0,1), \frac{\partial}{\partial t}^q \text{represents the Caputo fractional order derivative and } \int_0^1 \text{is the representation of the Riemann-Liouville fractional order integral, and are defined as } [24].
\end{align*}$$

Equation (1) could be applied in many real life applications. It is introduced in [25] to better characterise reaction diffusion processes in...